Impulse response matching estimators for DSGE models

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CMD estimation of macroeconomic models

- Impulse responses play a central role in macroeconomics.
- They can be estimated from VAR models ($\widehat{\gamma}_T$).
- Impulse responses can be obtained from DSGE models ($\gamma(\theta)$).
- Estimate θ by:

$$\widehat{\theta}_T \ = \ \text{argmin}_{\theta \in \Theta} (\widehat{\gamma}_T - \gamma(\theta))' W_T (\widehat{\gamma}_T - \gamma(\theta)).$$

- Rotemberg and Woodford (1997), Altig, Christiano, Eichenbaum and Lindé (2011), Boivin and Giannoni (2006), Christiano, Eichenbaum and Evans (2005), DiCecio (2005), DiCecio and Nelson (2007), Dupor, Han and Tsai (2007), Iacoviello (2005), Jordà and Kozicki (2007), Uribe and Yue (2006).

Minimum distance estimation of macroeconomic models

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• This paper considers another pitfall. In many applications, the $\sharp$ of impulse responses exceeds the $\sharp$ of the VAR parameters (e.g., Iacoviello 2005; Uribe and Yue 2006; Altig, Christiano, Eichenbaum and Lindé 2011).

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- This practice causes the joint distribution of the structural impulse responses to be asymptotically singular, and the standard asymptotic results no longer apply.

Outline of the talk

1. Asymptotic distributions of impulse response matching estimators and the test statistic of overidentifying restrictions

2. Asymptotic behavior of the Bayesian impulse response matching estimator

3. Confidence sets that are robust to weak identification

4. Monte Carlo simulation experiments

5. Empirical illustration

Impulse response matching estimators

Optimal weighting matrix:

$$\widehat{\theta}_{opt,T} \;=\; \text{argmin}_{\theta\in\Theta}(\widehat{\gamma}_T-\gamma(\theta))'\widehat{\Sigma}_T^{*-1}(\widehat{\gamma}_T-\gamma(\theta)),$$

where

$$\widehat{\Sigma}_T^{*} \;=\; \frac{1}{B}\sum_{b=1}^{B}(\widehat{\gamma}_T^{*(b)}-\bar{\widehat{\gamma}}_T^{*})(\widehat{\gamma}_T^{*(b)}-\bar{\widehat{\gamma}}_T^{*})'.$$

Diagonal weighting matrix:

$$\widehat{\theta}_{diag,T} \;=\; \text{argmin}_{\theta\in\Theta}(\widehat{\gamma}_T-\gamma(\theta))'W_T(\widehat{\gamma}_T-\gamma(\theta)),$$

where W_T is the diagonal matrix whose diagonal elements are the reciprocal of the diagonal elements of $\widehat{\Sigma}_T^{*}$.

Assumption (a)

$$Z_T \equiv \Omega^{-\frac{1}{2}}\sqrt{T}(\bar{X}_T - \mu) \quad \xrightarrow{d} \quad Z \equiv N(0, I_k),$$

$$Z_T^* \equiv \Omega^{-\frac{1}{2}}\sqrt{T}(\bar{X}_T^* - \bar{X}_T) \quad \xrightarrow{d} \quad Z^* \equiv N(0, I_k).$$

$AR(1)$ example

Suppose that

$$y_t \;=\; \rho y_{t-1} + u_t,$$

where $|\rho| < 1$ and $u_t \overset{iid}{\sim} (0, \sigma^2)$ with $E(u_t^3) = 0$ and $E(u_t^4) < \infty$ and that

$$\left[\begin{array}{c} T^{-\frac{1}{2}} \sum_{t=2}^{T} y_{t-1} u_t \\ T^{-\frac{1}{2}} \sum_{t=2}^{T} (y_{t-1}^2 - E(y_{t-1}^2)) \end{array}\right] \overset{d}{\to} N\left(\left[\begin{array}{c} 0 \\ 0 \end{array}\right], \left[\begin{array}{cc} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{array}\right] \right).$$

$AR(1)$ example

Let $Z_T = [Z_{1T}\ Z_{2T}]'$ where

$$\begin{aligned} Z_{1T} &= \frac{1}{\sqrt{\omega_{11} T}} \sum_{t=2}^{T} y_{t-1} u_t, \\ Z_{2T} &= \frac{1}{\kappa \sqrt{T}} \left[\sum_{t=2}^{T} (y_{t-1}^2 - E(y_{t-1}^2)) - \frac{\omega_{12}}{\omega_{11}} \sum_{t=2}^{T} y_{t-1} u_t \right], \end{aligned}$$

and $\kappa = (\omega_{22} - \omega_{12}^2/\omega_{22})^{1/2}$.

Then

$$Z_T \xrightarrow{d} N(0_{2\times 1}, I_2).$$

Assumption (a) is satisfied.

Assumption (b)

There are conformable matrices $B_0,\ldots,B_h$ and $\hat{B}_0,\ldots,\hat{B}_h$ such that

$$\begin{aligned} T^{\frac{1}{2}}(\widehat{\gamma}_T - \gamma) &= B_0 Z_T + T^{-\frac{1}{2}} B_1 (Z_T \otimes Z_T) + \ldots \\ &\quad + T^{-\frac{h}{2}} B_h (Z_T \otimes \cdots \otimes Z_T) + o_p(T^{-\frac{h}{2}}), \end{aligned}$$

$$\begin{aligned} T^{\frac{1}{2}}(\widehat{\gamma}_T^* - \widehat{\gamma}_T) &= \hat{B}_0 Z_T^* + T^{-\frac{1}{2}} \hat{B}_1 (Z_T^* \otimes Z_T^*) + \ldots \\ &\quad + T^{-\frac{h}{2}} \hat{B}_h (Z_T^* \otimes \cdots \otimes Z_T^*) + o_p^*(T^{-\frac{h}{2}}). \end{aligned}$$

where $\hat{B}_j = B_j + o_p(1)$ for $j = 0, 1, \ldots, h$.

$AR(1)$ example

By stochastic expansions,

$$
\begin{aligned}
& \widehat{\rho}_T - \rho \\
= \quad & \frac{\sum_{t=2}^T y_{t-1}u_t}{\sum_{t=2}^T y_{t-1}^2} \\
= \quad & \frac{1}{E(y_{t-1}^2)} \frac{1}{T} \sum_{t=2}^T y_{t-1}u_t - \frac{1}{[E(y_{t-1}^2)]^2} \frac{1}{T} \sum_{t=2}^T y_{t-1}u_t \frac{1}{T} \sum_{s=2}^T (y_{s-1}^2 - E(y_{s-1}^2)) + o_p\left(\frac{1}{T}\right) \\
= \quad & \frac{1-\rho^2}{\sigma^2} \frac{1}{T} \sum_{t=2}^T y_{t-1}u_t - \frac{(1-\rho^2)^2}{\sigma^4} \frac{1}{T} \sum_{t=2}^T y_{t-1}u_t \frac{1}{T} \sum_{s=2}^T (y_{s-1}^2 - E(y_{s-1}^2)) + o_p\left(\frac{1}{T}\right) \\
= \quad & \frac{1}{\sqrt{T}} (1-\rho^2)^{\frac{1}{2}} Z_{1T} + \frac{1}{T} \left(\frac{(1-\rho^2)^{\frac{3}{2}} \omega_{12}}{\sigma^2 \omega_{11}} Z_{1T}^2 + \frac{(1-\rho^2)^{\frac{3}{2}} \kappa}{\sigma^2} Z_{1T} Z_{2T} \right) + o_p\left(\frac{1}{T}\right).
\end{aligned}
$$

$AR(1)$ example

Because

$$\widehat{\rho}_T^2 - \rho^2 = 2\rho(\widehat{\rho}_T - \rho) + (\widehat{\rho}_T - \rho)^2,$$

we can write

$$\begin{aligned}
\left[T^{-\frac{1}{2}} \begin{array}{c} \widehat{\rho}_T - \rho \\ \widehat{\rho}_T^2 - \rho^2 \end{array} \right] &= \begin{bmatrix} (1-\rho^2)^{\frac{1}{2}} & 0 \\ 2\rho(1-\rho^2)^{\frac{1}{2}} & 0 \end{bmatrix} Z_T \\
&+ T^{-\frac{1}{2}} \begin{bmatrix} \frac{(1-\rho^2)^{\frac{3}{2}}\omega_{12}}{\sigma^2\omega_{11}} & \frac{(1-\rho^2)^{\frac{3}{2}}\kappa}{2\sigma^2} & \frac{(1-\rho^2)^{\frac{3}{2}}\kappa}{2\sigma^2} & 0 \\ 1-\rho^2+\frac{2\rho(1-\rho^2)^{\frac{3}{2}}\omega_{12}}{\sigma^2\omega_{11}} & \frac{\rho(1-\rho^2)^{\frac{3}{2}}\kappa}{\sigma^2} & \frac{\rho(1-\rho^2)^{\frac{3}{2}}\kappa}{\sigma^2} & 0 \end{bmatrix} (Z_T \otimes Z_T) \\
&+ o_p(T^{-1}) \\
&= B_0 Z_T + T^{-\frac{1}{2}} B_1 (Z_T \otimes Z_T) + o_p(T^{-\frac{1}{2}}).
\end{aligned}$$

Assumption (b) is satisfied.

Decomposition of the covariance matrix

By the Schur decomposition theorem, there exists an orthonormal matrix $\tilde{S}$ whose columns are eigenvectors of $B_0 B_0'$ and a diagonal matrix Λ whose diagonal elements are the eigenvalues of $B_0 B_0'$ such that

$$\tilde{S}' B_0 B_0' \tilde{S} \ = \ \Lambda.$$

Assumption (c)

We interchange the columns of $\widetilde{S}$ such that $S = [S_0 \; S_1 \; S_2 \; \cdots \; S_r]$, where S_0 is the $q \times q_0$ matrix that consists of the eigenvectors associated with the k largest eigenvalues of $B_0 B_0'$, and the matrices $S_1, S_2, ..., S_r$ are of dimension $q \times q_1$, $q \times q_2, ...,$ $q \times q_r$, respectively, such that $q_0 = k$ and $q_0 + q_1 + ... + q_r = q$. Let $S_0, S_1, ..., S_r$ satisfy the conditions: (i) $S_i' B_j = 0$ for $\forall i = 1, ..., r$ and $\forall j < j_i$; and (ii)

$$\xi = \begin{bmatrix} S_0' B_0 Z \\ S_1' B_{j_1} (Z \otimes \cdots \otimes Z) \\ \vdots \\ S_r' B_{j_r} (Z \otimes \cdots \otimes Z) \end{bmatrix}$$

has a nonsingular second moment matrix $J = E(\xi \xi')$, where $j_1, j_2, ..., j_r$ are integers such that $0 < j_1 < j_2 < ... < j_r$.

$AR(1)$ example

By the Schur decomposition theorem, we have

$$S' B_0 B_0' S = \Lambda,$$

where

$$\begin{aligned} S &= \frac{1}{(4\rho^2+1)^{\frac{1}{2}}} \begin{bmatrix} 1 & 2\rho \\ 2\rho & -1 \end{bmatrix} \equiv \begin{bmatrix} S_0 & S_1 \end{bmatrix}, \\ B_0 &= \begin{bmatrix} (1-\rho^2)^{\frac{1}{2}} & 0 \\ 2\rho(1-\rho^2)^{\frac{1}{2}} & 0 \end{bmatrix}, \\ \Lambda &= \begin{bmatrix} (1-\rho^2)(4\rho^2+1) & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

$AR(1)$ example

$$\xi = \begin{bmatrix} S_0' B_0 Z \\ S_1' B_1 (Z \otimes Z) \end{bmatrix}$$

has a nonsingular second moment matrix:

$$\begin{bmatrix} (1-\rho^2)(4\rho^2+1) & 0 \\ 0 & \frac{3(1-\rho^2)^2}{4\rho^2+1} \end{bmatrix}.$$

Assumption (c) is satisfied.

Consistency of impulse response matching estimators

- Because $\widehat{\gamma}_T$ is consistent for γ and $W = \text{plim}\, W_T$ is positive definite, it is relatively staightforward to show that $\widehat{\theta}_{diag,T}$ is consistent for θ_0.

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- Because $\widehat{\gamma}_T$ is consistent for γ and $W = \text{plim} W_T$ is positive definite, it is relatively staightforward to show that $\widehat{\theta}_{diag,T}$ is consistent for θ_0.

- Because

$$
\begin{aligned}
& (\widehat{\gamma}_T - f(\theta))' \widehat{\Sigma}_T^{*-1} (\widehat{\gamma}_T - f(\theta)) \\
= \; & [\Upsilon_T S_T' (\widehat{\gamma}_T - f(\theta))]' (\Upsilon_T S_T' \widehat{\Sigma}_T S_T \Upsilon_T)^{-1} [\Upsilon_T S_T' (\widehat{\gamma}_T - f(\theta))] \\
\approx \; & (\xi + \Upsilon_T S_T' (f(\theta_0) - f(\theta)))' J^{-1} (\xi + \Upsilon_T S_T' (f(\theta_0) - f(\theta)))
\end{aligned}
$$

$\widehat{\theta}_{opt,T}$ is also consistent for θ_0.

$AR(1)$ example

Let

$$\Upsilon_T = \begin{bmatrix} T^{\frac{1}{2}} & 0 \\ 0 & T \end{bmatrix}.$$

Then

$$\begin{aligned}
& \Upsilon_T S' \begin{bmatrix} \widehat{\rho}_T - \rho \\ \widehat{\rho}_T^2 - \rho^2 \end{bmatrix} \\
= & \begin{bmatrix} [(1-\rho^2)(4\rho^2+1)]^{\frac{1}{2}} & 0 \\ 0 & 0 \end{bmatrix} Z_T \\
& + \begin{bmatrix} \frac{1}{\sqrt{T}} \frac{2\rho(1-\rho^2)}{(4\rho^2+1)^{\frac{1}{2}}} + \frac{1}{\sqrt{T}} \frac{(4\rho^2+1)^{\frac{1}{2}}(1-\rho^2)^{\frac{3}{2}}\omega_{12}}{\sigma^2\omega_{11}} & \frac{(4\rho^2+1)^{\frac{1}{2}}(1-\rho^2)^{\frac{3}{2}}\kappa}{2\sigma^2 T^{\frac{1}{2}}} & \frac{(4\rho^2+1)^{\frac{1}{2}}(1-\rho^2)^{\frac{3}{2}}\kappa}{2\sigma^2 T^{\frac{1}{2}}} & 0 \\ -\frac{1-\rho^2}{(4\rho^2+1)^{\frac{1}{2}}} & 0 & 0 & 0 \end{bmatrix} \\
& \times (Z_T \otimes Z_T) + o_p(1) \\
= & \begin{bmatrix} [(1-\rho^2)(4\rho^2+1)]^{\frac{1}{2}} Z_{1T} \\ -\frac{1-\rho^2}{(4\rho^2+1)^{\frac{1}{2}}} Z_{1T}^2 \end{bmatrix} + o_p(1),
\end{aligned}$$

Asymptotic distributions of impulse response matching estimators

- The asymptotic distribution of $\widehat{\theta}_{diag,T}$ depends on the first term in $T^{\frac{1}{2}}(\widehat{\gamma}_T - f(\theta_0))$ $(B_0 Z)$ only and is normal.

Asymptotic distributions of impulse response matching estimators

- The asymptotic distribution of $\widehat{\theta}_{diag,T}$ depends on the first term in $T^{\frac{1}{2}}(\widehat{\gamma}_T - f(\theta_0))$ $(B_0 Z)$ only and is normal.

- The asymptotic distribution of $\widehat{\theta}_{opt,T}$ depends not only on the first term but also on higher order terms in $T^{\frac{1}{2}}(\widehat{\gamma}_T - f(\theta_0))$ and is nonnormal.

Asymptotic distributions of impulse response matching estimators

Suppose that Assumptions (a)–(g) hold. Then
(a)

$$\begin{aligned} T^{\frac{r+1}{2}}(\widehat{\theta}_{opt,T} - \theta_0) &\xrightarrow{d} (F_0' S_r J^{rr-1} S_r' F_0)^{-1} F_0' S_r J^{r-1} \xi, \\ T^{\frac{r+1}{2}}(\widehat{\theta}^*_{opt,T} - \widehat{\theta}_{opt,T}) &\xrightarrow{d} (F_0' S_r J^{rr-1} S_r' F_0)^{-1} F_0' S_r J^{r-1} \xi^*, \end{aligned}$$

where J^{rr-1} is the $q_r \times q_r$ lower-right submatrix of J^{-1}, and J^{r-1} is the $q_r \times q$ lower submatrix of J^{-1}.

Asymptotic distributions of impulse response matching estimators

(b)

$$T^{\frac{1}{2}}(\widehat{\theta}_{diag,T} - \theta_0) \quad \xrightarrow{d} \quad (F_0' W F_0)^{-1} F_0' W B_0 Z,$$

$$T^{\frac{1}{2}}(\widehat{\theta}^*_{diag,T} - \widehat{\theta}_{diag,T}) \quad \xrightarrow{d} \quad (F_0' W F_0)^{-1} F_0' W B_0 Z^*.$$

The J test of overidentifying restrictions

To test $\gamma(\mu) = f(\theta_0)$, consider

$$J_T = (\widehat{\gamma}_T - f(\widehat{\theta}_{opt,T}))' \widehat{\Sigma}_T^{*-1} (\widehat{\gamma}_T - f(\widehat{\theta}_{opt,T})).$$

The bootstrap version of this test statistic is defined as

$$\begin{aligned} J_T^* &= \left(\widehat{\gamma}_T^* - f(\widehat{\theta}_{opt,T}^*) - \widehat{\gamma}_T + f(\widehat{\theta}_{opt,T})\right)' \widehat{\Sigma}_T^{**-1} \\ &\quad \times \left(\widehat{\gamma}_T^* - f(\widehat{\theta}_{opt,T}^*) - \widehat{\gamma}_T + f(\widehat{\theta}_{opt,T})\right), \end{aligned}$$

where the term $\widehat{\gamma}_T - f(\widehat{\theta}_{opt,T})$ accomplishes the required recentering (Hall and Horowitz 1996).

Asymptotic distribution of the test statistic for overidentifying restrictions

Suppose that Assumptions (a)–(g) and (h)(1) hold. Then

$$\begin{aligned} J_T &\xrightarrow{d} \eta'\eta, \\ J_T^* &\xrightarrow{d} \eta^{*\prime}\eta^*, \end{aligned}$$

where

$$\begin{aligned} \eta &= J^{-\frac{1}{2}}\xi - J^{-\frac{1}{2}}S_r'F_0(F_0'S_rJ^{rr-1}S_r'F_0)^{-1}F_0'S_rJ^{r-1}\xi, \\ \eta^* &= J^{-\frac{1}{2}}\xi^* - J^{-\frac{1}{2}}S_r'F_0(F_0'S_rJ^{rr-1}S_r'F_0)^{-1}F_0'S_rJ^{r-1}\xi^*. \end{aligned}$$

Bayesian impulse response matching

• Bayesian impulse response matching estimators have been used in Christiano, Trabandt and Walentin (2011), Christiano, Eichenbaum and Trabandt (2013), and Kormilitsina and Nekipulov (2013).

• The quasi-posterior density is defined as

$$p(\theta) = \frac{\exp(-q_T(\theta))\pi(\theta)}{\int_\Theta \exp(-q_T(\theta))\pi(\theta)d\theta},$$

where $\pi(\theta)$ is the prior density and

$$q_T(\theta) = \frac{1}{2}(\widehat{\gamma}_T - f(\theta))'\widehat{\Sigma}_T^{*-1}(\widehat{\gamma}_T - f(\theta)).$$

Under the standard assumptions the quasi-posterior density converges to the asymptotic distribution of the impulse response matching estimator (Chernozhukov and Hong, 2003).

Asymptotic behavior of the quasi-posterior distribution

Suppose that Assumptions (a)–(g) and (h)(1) hold. Then, using the notation of Chernozhykov and Hong (2003),

$$\|p_T^*(h)-p_\infty^*(h)\|_{TVM(\alpha)} \equiv \int_{H_T}(1+\|h\|^\alpha)|p_T^*(h)-p_\infty^*(h)|dh \;=\; o_p^*(1)$$

in prob-P, where

$$\begin{aligned} H_T &= \{h \in \Re^l : h = T^{\frac{j_r+1}{2}}(\theta-\theta_0) - T^{\frac{j_r+1}{2}}(\nabla^2 q_T(\theta_0))^{-1}\nabla q_T(\theta_0) \\ &\quad \text{for some } \theta \in \Theta\}, \\ p_\infty^*(h) &= \sqrt{\frac{|T^{-(j_r+1)}\nabla^2 q_T(\theta_0)|}{(2\pi)^l}} \exp\left(-\frac{1}{2T^{j_r+1}} h'\nabla^2 q_T(\theta_0) h\right). \end{aligned}$$

Asymptotic behavior of the quasi-posterior

- While the quasi-posterior distribution converges to a normal distribution, the posterior mean converges in distribution to the distributions discussed earlier.

- Even when the optimal weighting matrix is used, the standard errors cannot be obtained from MCMC draws.

- One can use the bootstrap to construct confidence intervals, however.

Weak identification

- The structural parameters of DSGE models may not be strongly identified (e.g., Canova and Sala, 2009; Guerron-Quintana, Inoue and Kilian 2013; Dufour, Khalaf and Kichian 2013; Qu 2014; Andrews and Mikusheva 2015).

- Building on the earlier results, we propose a nonstandard confidence interval for structural parameters that is robust to weak identification.

Asymptotic distributions of the Wald test statistic

Suppose that Assumptions (a)–(g) hold. Under $H_0 : \gamma(\mu) = f(\theta_0)$ for some $\theta_0 \in \Theta$, where θ_0 need not be unique,

$$\mathcal{W}_T \overset{d^*}{\to} \xi' J^{-1} \xi \text{ in prob-}P,$$
$$\mathcal{W}_T^* \overset{d^{**}}{\to} \xi^{*\prime} J^{-1} \xi^* \text{ in prob-}P^*,$$

where

$$\begin{aligned}
\mathcal{W}_T &= (\widehat{\gamma}_T - f(\theta_0))' \Sigma_T^{*-1} (\widehat{\gamma}_T - f(\theta_0)), \\
\mathcal{W}_T^* &= (\widehat{\gamma}_T^* - \widehat{\gamma}_T)' \Sigma_T^{**-1} (\widehat{\gamma}_T^* - \widehat{\gamma}_T), \\
\Sigma_T^* &= \frac{1}{B} \sum_{j=1}^{B} \left(\widehat{\gamma}_T^{*(j)} - \bar{\widehat{\gamma}}_T^* \right) \left(\widehat{\gamma}_T^{*(j)} - \bar{\widehat{\gamma}}_T^* \right)', \\
\Sigma_T^{**(j)} &= \frac{1}{B} \sum_{k=1}^{B} \left(\widehat{\gamma}_T^{**(j,k)} - \bar{\widehat{\gamma}}_T^{(j)**} \right) \left(\widehat{\gamma}_T^{**(j,k)} - \bar{\widehat{\gamma}}_T^{(j)**} \right)'
\end{aligned}$$

The data-generating process

Consider

$$\begin{aligned}
\pi_t &= \kappa x_t + \beta \mathbb{E}(\pi_{t+1}|\mathcal{I}_{t-1}), \\
R_t &= \rho_r R_{t-1} + (1-\rho_r)\phi_\pi \pi_t + (1-\rho_r)\phi_x x_t + \xi_t, \\
x_t &= \mathbb{E}(x_{t+1}|\mathcal{I}_{t-1}) - \sigma\left(\mathbb{E}(R_t|\mathcal{I}_{t-1}) - \mathbb{E}(\pi_{t+1}|\mathcal{I}_{t-1}) - z_t\right), \\
z_t &= \rho_z z_{t-1} + \sigma^z \varepsilon_t^z, \\
\xi_t &= \sigma^r \varepsilon_t^r,
\end{aligned}$$

where $\sigma = 1$, $\alpha = 0.75$, $\beta = 0.99$, $\phi_\pi = 1.5$, $\phi_x = 0.125$, $\omega = 1$, $\rho_r = 0.75$, $\rho_z = 0.90$, $\theta = 6$, $\sigma^z = 0.30$, and $\sigma^r = 0.20$.

Bias, SD and RMSE ($T = 232$)

		Diagonal weighting matrix			Optimal weighting matrix		
p	H	Mean bias	Standard deviation	RMSE	Mean bias	Standard deviation	RMSE
2	2	0.0022	0.0103	0.0105	0.0012	0.0073	0.0074
2	4	0.0030	0.0122	0.0126	0.0011	0.0073	0.0074
2	8	0.0043	0.0145	0.0151	0.0015	0.0076	0.0078
4	2	0.0029	0.0100	0.0104	0.0020	0.0075	0.0078
4	4	0.0034	0.0115	0.0120	0.0021	0.0074	0.0077
4	8	0.0045	0.0137	0.0144	0.0021	0.0075	0.0078
6	2	0.0038	0.0100	0.0107	0.0028	0.0076	0.0081
6	4	0.0044	0.0113	0.0121	0.0028	0.0076	0.0081
6	8	0.0053	0.0132	0.0143	0.0028	0.0077	0.0081

Effective coverage probabilities of 90% CI ($T = 232$)

		VAR bootstrap		DSGE bootstrap	
p	H	Diagonal	Optimal	Diagonal	Optimal
2	2	86.4	84.6	87.2	89.6
2	4	85.8	75.8	87.8	91.2
2	8	87.0	67.6	86.4	90.2
4	2	86.2	87.6	88.0	88.6
4	4	85.6	87.0	87.0	88.8
4	8	87.0	74.8	86.2	89.4
6	2	88.0	86.2	89.0	89.0
6	4	85.6	87.8	87.0	90.2
6	8	87.0	81.6	86.8	87.4

Effective coverage probabilities of 90% identification-robust CI ($T = 232$)

p	H	VAR bootstrap	DSGE bootstrap
2	2	92.2	90.6
2	4	91.6	91.8
2	8	90.0	91.8
4	2	91.8	91.4
4	4	92.0	90.4
4	8	90.0	91.8
6	2	93.0	90.0
6	4	92.8	89.0
6	8	93.4	91.2

Estimation of medium-scale DSGE models

- Medium-scale New Keynesian DSGE models (Christiano, Eichenbaum and Evans, 2005; Smets and Wouters, 2007; Altig, Christiano, Eichenbaum and Lindé, 2011).
- VAR(2) model consists of 13 variables.
- Sample period: from 1951Q1 to 2008Q4.
- Impulse responses of these variables to three structural shocks: the monetary policy shock, the shock to the neutral productivity growth, and the shock to the investment-specific technology growth.
- Short-run and long-run restrictions to identify structural impulse responses.

Estimates for medium-scale DSGE model (H=15)

	Frequentist		Bayesian						CTW	
		s.e.	mode	s.e.	med.	s.e.	mean	s.e.	mean	s.e.
Price stickiness	0.660	0.068	0.623	0.078	0.623	0.077	0.622	0.077	0.622	0.036
Std. monetary policy	0.592	0.077	0.504	0.077	0.509	0.077	0.509	0.077	0.509	0.044
Std. neutral tech.	0.228	0.059	0.224	0.056	0.225	0.056	0.226	0.055	0.226	0.016
Autocorr. invest. tech.	0.617	0.105	0.620	0.096	0.607	0.102	0.600	0.104	0.600	0.071
Std. invest. tech.	0.152	0.038	0.158	0.040	0.159	0.040	0.160	0.040	0.160	0.022
Taylor rule: Interest smoothing	0.820	0.054	0.880	0.068	0.879	0.068	0.879	0.069	0.879	0.015
Taylor rule: Inflation	1.030	0.365	1.449	0.862	1.462	0.870	1.472	0.874	1.472	0.113
Taylor rule: Output gap	0.0005	0.041	0.087	0.187	0.091	0.196	0.095	0.202	0.095	0.033
Invest. adjustment costs	16.10	8.09	12.44	6.21	13.03	6.56	13.32	6.74	13.32	2.70
Consumption habit	0.757	0.071	0.763	0.064	0.762	0.065	0.761	0.065	0.761	0.018
Capacity adjustment costs	0.156	0.189	0.296	0.269	0.312	0.286	0.318	0.290	0.318	0.081
Price markup	1.250	0.215	1.178	0.210	1.189	0.217	1.194	0.221	1.194	0.078
Inv. labor supply elasticity	0.146	0.135	0.106	0.092	0.108	0.095	0.111	0.098	0.111	0.023

Estimates for medium-scale DSGE model (H=15)

	Frequentist		Bayesian						CTW	
		s.e.	mode	s.e.	med.	s.e.	mean	s.e.	mean	s.e.
Price stickiness	0.660	0.068	0.623	0.078	0.623	0.077	0.622	0.077	0.622	0.036
Std. monetary policy	0.592	0.077	0.504	0.077	0.509	0.077	0.509	0.077	0.509	0.044
Std. neutral tech.	0.228	0.059	0.224	0.056	0.225	0.056	0.226	0.055	0.226	0.016
Autocorr. invest. tech.	0.617	0.105	0.620	0.096	0.607	0.102	0.600	0.104	0.600	0.071
Std. invest. tech.	0.152	0.038	0.158	0.040	0.159	0.040	0.160	0.040	0.160	0.022
Taylor rule: Interest smoothing	0.820	0.054	0.880	0.068	0.879	0.068	0.879	0.069	0.879	0.015
Taylor rule: Inflation	1.030	0.365	1.449	0.862	1.462	0.870	1.472	0.874	1.472	0.113
Taylor rule: Output gap	0.0005	0.041	0.087	0.187	0.091	0.196	0.095	0.202	0.095	0.033
Invest. adjustment costs	16.10	8.09	12.44	6.21	13.03	6.56	13.32	6.74	13.32	2.70
Consumption habit	0.757	0.071	0.763	0.064	0.762	0.065	0.761	0.065	0.761	0.018
Capacity adjustment costs	0.156	0.189	0.296	0.269	0.312	0.286	0.318	0.290	0.318	0.081
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The 95% confidence intervals for the length of a price contract include Klenow and Kryvtsov's (2008) estimate based on micro data (2.3 quarters).

Estimates for medium-scale DSGE model (H=19)

	Frequentist		Bayesian						CTW	
		s.e.	mode	s.e.	med.	s.e.	mean	s.e.	mean	s.e.
Price stickiness	0.744	0.063	0.711	0.074	0.707	0.076	0.704	0.077	0.704	0.040
Std. monetary policy	0.596	0.074	0.552	0.078	0.551	0.078	0.551	0.077	0.551	0.043
Std. neutral tech.	0.331	0.090	0.293	0.084	0.293	0.084	0.293	0.084	0.293	0.028
Autocorr. invest. tech.	0.678	0.075	0.663	0.098	0.649	0.103	0.643	0.105	0.643	0.060
Std. invest. tech.	0.143	0.044	0.148	0.045	0.152	0.045	0.154	0.044	0.154	0.021
Taylor rule: Interest smoothing	0.868	0.056	0.887	0.075	0.885	0.077	0.885	0.078	0.885	0.014
Taylor rule: Inflation	1.260	0.464	1.464	0.890	1.479	0.907	1.488	0.914	1.488	0.110
Taylor rule: Output gap	0.175	0.166	0.188	0.298	0.200	0.319	0.206	0.329	0.206	0.060
Invest. adjustment costs	6.570	3.114	7.750	4.222	8.069	4.472	8.298	4.639	8.298	1.888
Consumption habit	0.739	0.071	0.748	0.067	0.746	0.069	0.746	0.069	0.746	0.020
Capacity adjustment costs	0.370	0.270	0.340	0.246	0.355	0.267	0.362	0.276	0.362	0.080
Price markup	1.250	0.218	1.166	0.195	1.174	0.203	1.180	0.207	1.180	0.074
Inverse labor supply elasticity	0.0548	0.051	0.062	0.060	0.066	0.064	0.069	0.066	0.069	0.017

Estimates for medium-scale DSGE model (H=19)

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Inverse labor supply elasticity	0.0548	0.051	0.062	0.060	0.066	0.064	0.069	0.066	0.069	0.017

The 95% confidence intervals based on the frequentist and CTW exclude 2.3 whereas our 95% confidence intervals include it.

Allowing for weak identification

		$H = 15$		$H = 19$	
Price Stickiness	ξ_p	0.460	0.726	0.579	0.796
Std. Monetary Policy Shock	$\sigma_{\epsilon,r}$	0.380	0.664	0.385	0.690
Std. Neutral Tech. Shock	$\sigma_{\epsilon,\mu}$	0.182	0.299	0.211	0.374
Autocorr. Invest. Tech. Shock	$\rho_{\mu,\psi}$	0.330	0.764	0.437	0.792
Std. Invest. Tech. Shock	$\sigma_{\epsilon,\psi}$	0.108	0.231	0.101	0.239
Taylor Rule: Interest Smoothing	ρ_r	0.840	0.922	0.841	0.921
Taylor Rule: Inflation	ϕ_π	1.210	1.912	1.241	1.915
Taylor Rule: Output Gap	ϕ_y	0.018	0.262	0.063	0.425
Investment Adjustment Costs	S''	6.902	26.245	4.327	18.819
Consumption Habit	b	0.702	0.815	0.681	0.811
Capacity Adjustment Costs	σ_a	0.078	0.683	0.155	0.632
Price Markup	λ_p	1.017	1.502	1.015	1.464
Inverse Labor Supply Elasticity	υ	0.056	0.227	0.027	0.162

Concluding remarks

- We propose a bootstrap approach to inference about impulse response matching estimators that allows including more impulse responses than VAR parameters.

- The baseline results assume strong identification. We extended this method to allow for possible weak identification of the structural model parameters.

- Inference based on quasi-Bayesian estimators requires sandwich estimator of variance.

Future research

• Key question: How to choose p and h (also see Hall, Inoue, Nason and Rossi, 2012)

• We focused on stationary models. Extensions to possibly integrated and possibly cointegrated models are possible based on lag-augmented VAR models (see Dolado and Lütkepohl 1996; Toda and Yamamoto 1995).